PART - I (CHEMISTRY)

SECTION–I : (Single Correct Choice Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

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1. The synthesis of 3–octyne is achieved by adding a bromoalkane into a mixture of sodium amide and an alkyne. The bromoalkane and alkyne respectively are –

(A) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3CH_2C{\equiv}CH$ (B) $BrCH_2CH_2CH_3$ and $CH_3CH_2CH_2C{\equiv}CH$

(C) $BrCH_2CH_2CH_2CH_2CH_3$ and $CH_3C{\equiv}CH$ (D) $BrCH_2CH_2CH_2CH_3$ and $CH_3CH_2C{\equiv}CH$

Ans. (D)

Sol. Since the target molecule is

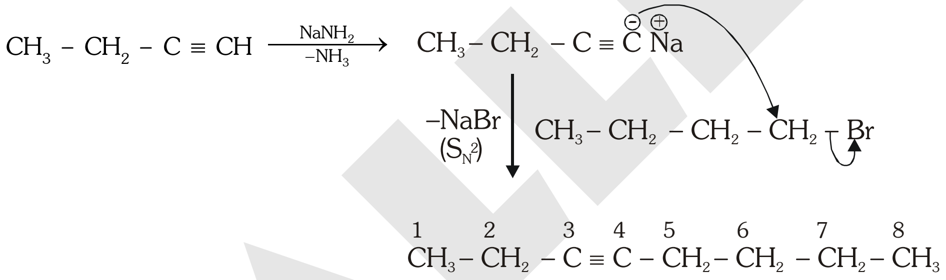
$\underset{1}{CH_3}-\underset{2}{CH_2}-\underset{3}{C}\equiv\underset{4}{C}-\underset{5}{CH_2}-\underset{6}{CH_2}-\underset{7}{CH_2}-\underset{8}{CH_3}$

There are two possibilities for the formation of product by reaction of alkynide anion and alkyl halide.

Reaction of $CH_3\,CH_2CH_2\,C \equiv C^{\ominus}$ and $Br-CH_2-CH_2-CH_3$ Ist possibility

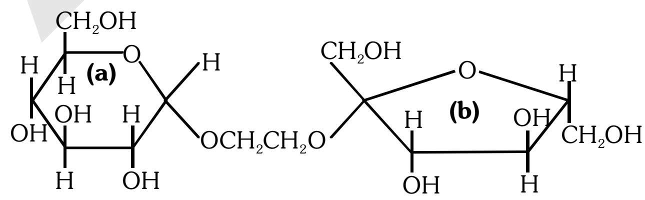
Reaction of $CH_3\,CH_2\,C \equiv C^{\ominus}$ and $Br-CH_2-CH_2-CH_2-CH_3$ IInd possibility

so correct answer is (D)



3-Octyne

2. The correct statement about the following disaccharide is –

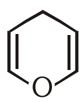


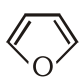
(A) Ring **(a)** is pyranose with α–glycosidic link (B) Ring **(a)** is furanose with α–glycosidic link

(C) Ring **(b)** is furanose with α–glycosidic link (D) Ring **(b)** is pyranose with β–glycosidic link

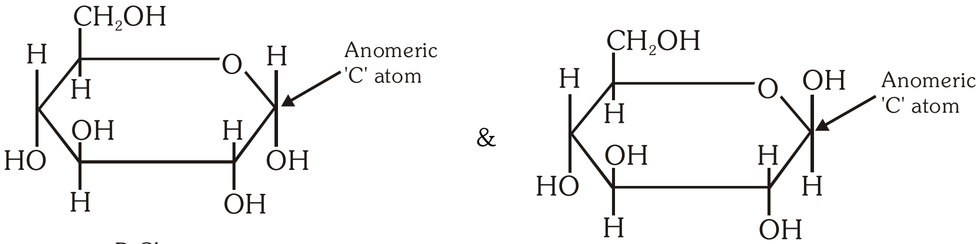
Ans. (A)

In cyclic structure of glucose, 2 kinds of ring structures are possible.

(1)  → pyran

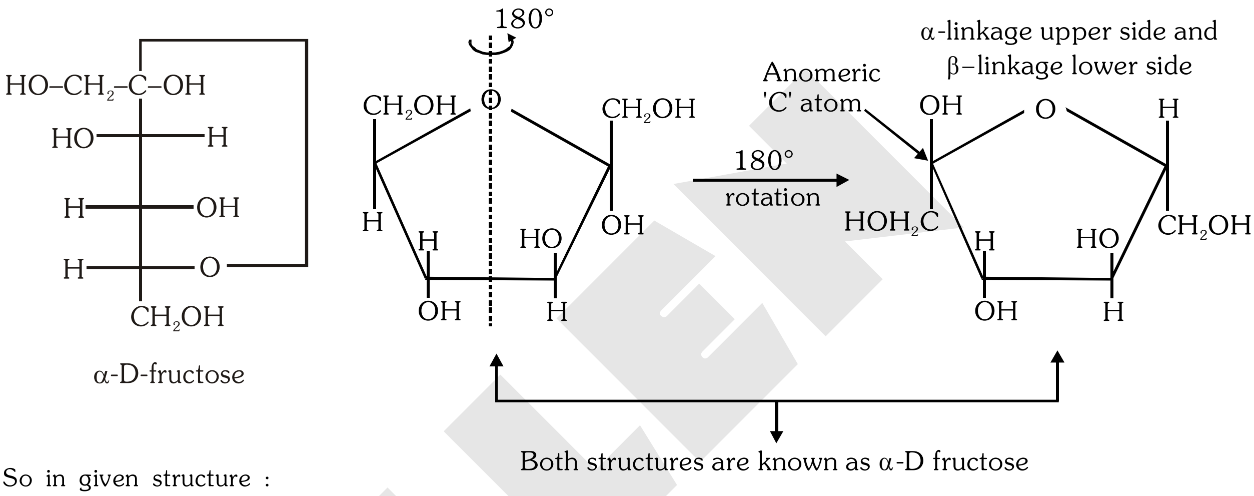
(2)  → furan

In Haworth projection α–D glucose and β–D glucose are represented by following structures.

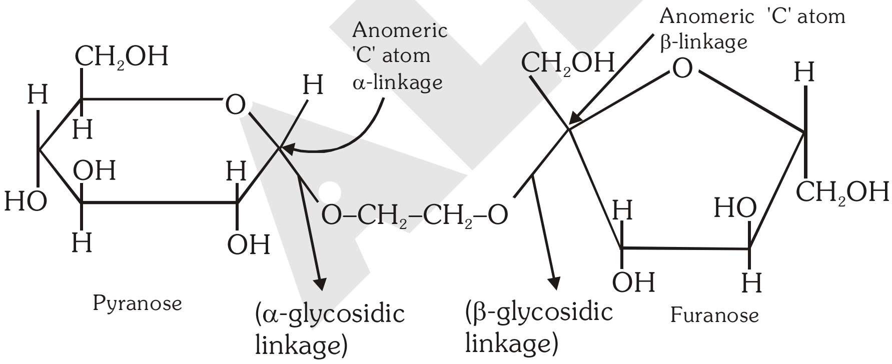


α-D Glucose β-D Glucose

In fructose, 2 structures of α–D fructose are :

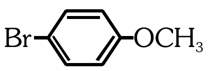


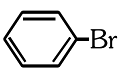
So in given structure :

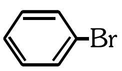


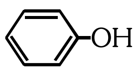
So in above structure ring (a) is pyranose with α-glycosidic linkage.

3. In the reaction  the products are :-

(A)  and H_2

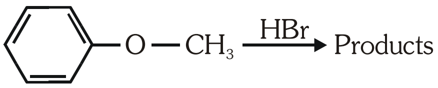
(B)  and CH_3Br

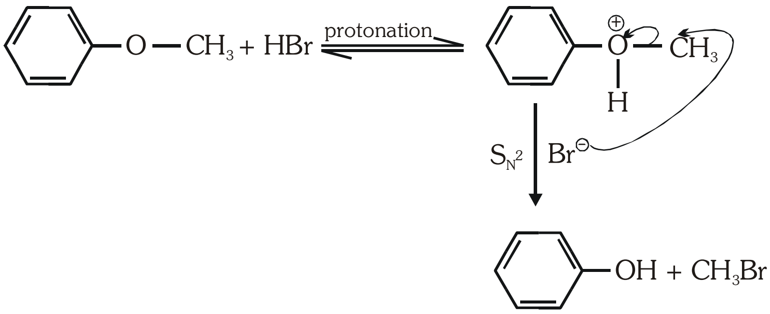
(C)  and CH_3OH

(D)  and CH_3Br

Ans. **(D)**

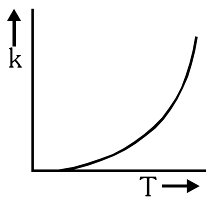
Sol. For the given reaction

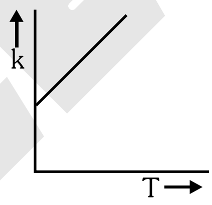


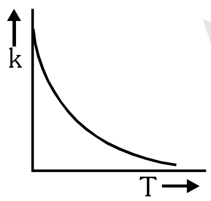


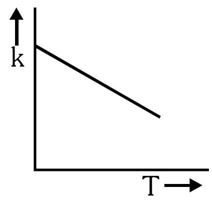
The attack of $Br^{\ominus}$ takes place from the back side to $-CH_3$, as the attack to ring carbon cannot takes place by S_{N^1} or S_{N^2} mechanism.

4. Plots showing the variation of the rate constant (k) with temperature (T) are given below. The plot that follows Arrhenius equation is –

(A)


(B)


(C)


(D)


Ans. (A)

Sol. **Arrhenius equation :** $k = Ae^{-E_a/RT}$

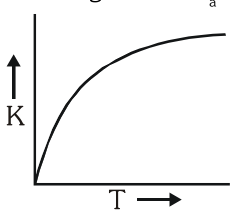
(taking A and E_a to be constant, differentiating w.r.t. T)

$$\frac{dk}{dT} = \left[\frac{AE_a}{RT^2}\right]e^{-E_a/RT}$$

as $T \to 0 \Rightarrow$ slope $= \frac{dK}{dT} \to \infty$

as $T \to \infty \Rightarrow$ slope $= \frac{dK}{dT} \to 0$

assuming A and E_a to be constant theoretically, the plot should be



But $\because$ No such option is given. Now experimentally, A and E_a both vary with temperature and k increases as T increases and become very large at infinite temperature. Hence option (A) is correct.

5. The species which by definition has **ZERO** standard molar enthalpy of formation at 298 K is –

(A) $Br_2(g)$ (B) $Cl_2(g)$ (C) $H_2O(g)$ (D) $CH_4(g)$

Ans. (B)

Sol. $Cl_2(g)$ present is its natural most stable form.

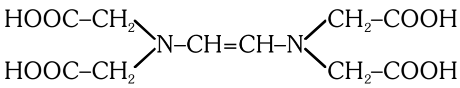
6. The bond energy (in **kcal mol^{-1}**) of a C–C single bond is approximately –

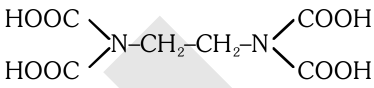
(A) 1 (2) 10 (C) 100 (D) 1000

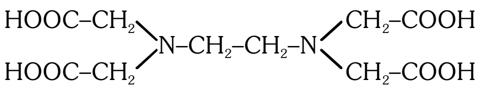
Ans. (C)

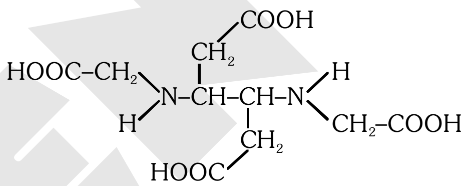
Ans. The approximate C–C single bond energy is 100 Kcal mol^{-1}.

7. The correct structure of ethylenediaminetetraacetic acid (EDTA) is –

(A) 

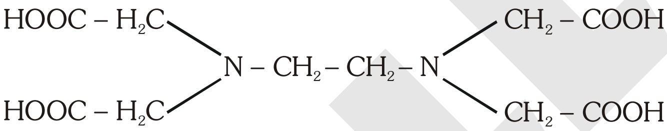
(B) 

(C) 

(D) 

Ans. (C)

Sol. The correct structure of ethylenediaminetetra acetic acid (EDTA) is



8. The ionization isomer of $[Cr(H_2O)_4Cl(NO_2)]Cl$ is –

(A) $[Cr(H_2O)_4(O_2N)]Cl_2$ (B) $[Cr(H_2O)_4Cl_2](NO_2)$ (C) $[Cr(H_2O)_4Cl(ONO)]Cl$ (D) $[Cr(H_2O)_4Cl_2(NO_2)].H_2O$

Ans. (B)

Sol. Ionisation isomers differ in ions in solution thus, ionisation isomer of $[Cr(H_2O)_4Cl(NO_2)]Cl$ is $[Cr(H_2O)_4Cl_2]$ (NO_2). Because

$[Cr(H_2O)_4Cl(NO_2)]Cl \longrightarrow [Cr(H_2O)_4Cl(NO_2)]^+ + Cl^-$

(Given compound)

$[Cr(H_2O)_4Cl_2]$ $(NO_2) \longrightarrow [Cr(H_2O)_4Cl_2]^+ + NO_2^-$

Ionisation isomer of given compound.

SECTION–II : (Multiple Correct Choice Type)

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

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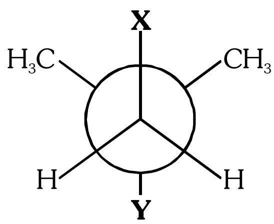
9. In the Newman projection for 2,2–dimethylbutane



X and **Y** can respectively be –

(A) H and H (B) H and C_2H_5 (C) C_2H_5 and H (D) CH_3 and CH_3

Ans. (B, D)

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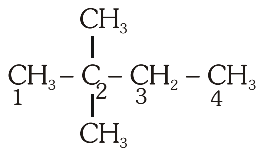
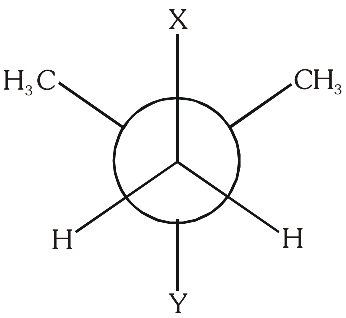
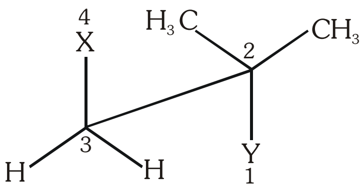
Sol. Given molecule 2, 2 - dimethylbutane, 

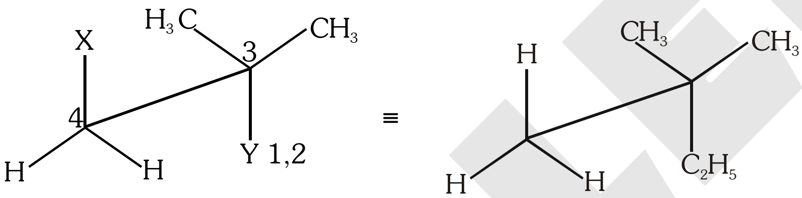
Given newman projection is :



Can be represented in sawhorse projection-



Group X : – CH_3
Y : – CH_3

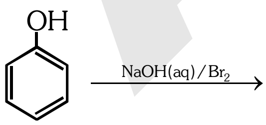


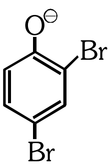
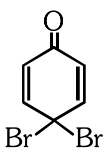
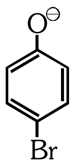
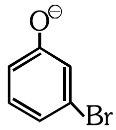
Group X : – H
Y : – C_2H_5

10. Aqueous solutions of HNO_3, KOH, CH_3COOH and CH_3COONa of identical concentrations are provided. The pair(s) of solutions which form a buffer upon mixing is(are) –

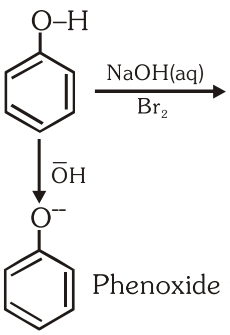
(A) HNO_3 and CH_3COOH
(B) KOH and CH_3COONa
(C) HNO_3 and CH_3COONa
(D) CH_3COOH and CH_3COONa

Ans. (C, D)

11. In the reaction  the intermediate(s) is(are) –

(A) 
(B) 
(C) 
(D) 

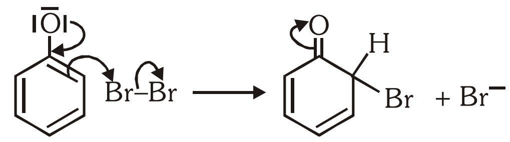
Ans. (A, B, C)

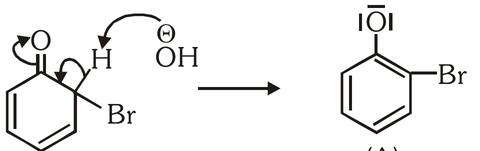


Phenoxide ion is powreful activating & o/p directing

The various intermedaites are formed as shown :

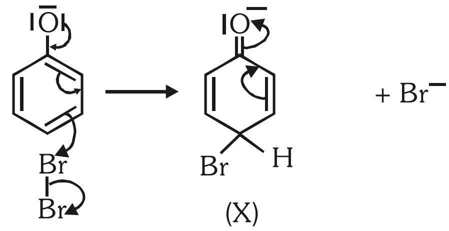
(I)



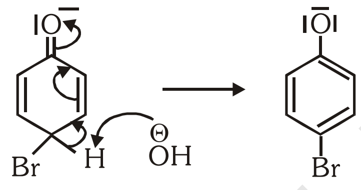


(A)

(II)

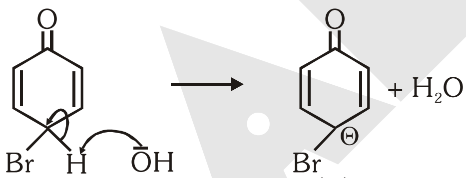


(X)



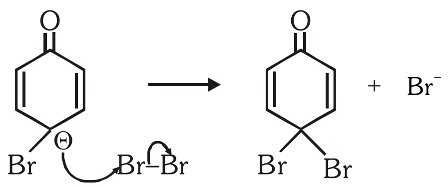
(X) (C)

(III) If the intermediate X reacts with 2nd mole of $\overset{\ominus}{O}H$



(Y)

The intermediate Y can react with 2nd mole of Br_2 to give (B) as



The option (B) can only be predicted by an expert in subject. Such indepth approach do not seems to be the general tone of the paper. Still we prefer the option (B) to be correct.

12. The reagent(s) used for softening the temporary hardness of water is(are) –

(A) $Ca_3(PO_4)_2$ (B) $Ca(OH)_2$ (C) Na_2CO_3 (D) NaOCl

Ans. (B, C)

Sol. Temporary hardness is due to presence of Ca^{2+} and Mg^{2+} ions in the form of their soluble bicarbonate salt (i.e. $Ca(HCO_3)_2$ and $Mg(HCO_3)_2$). Temporary hardness can be removed by $Ca(OH)_2$ as

$Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + 2H_2O$

$Mg(HCO_3)_2 + Ca(OH)_2 \longrightarrow CaCO_3\downarrow + MgCO_3 + 2H_2O$

Na_2CO_3 can be used to remove both temporary as well as permanent hardness.

$Ca(HCO_3)_2 + Na_2CO_3 \longrightarrow CaCO_3\downarrow + 2NaHCO_3$

$Mg(HCO_3)_2 + Na_2CO_3 \longrightarrow MgCO_3\downarrow + 2NaHCO_3$

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13. Among the following, the intensive property is (properties are) –

(A) Molar conductivity (B) Electromotive force (C) Resistance (D) Heat capacity

Ans. (A, B)

Sol. $\because \ \Lambda_M = \dfrac{\text{conductance}}{\text{mole}}$

$\because \ \text{EMF} = \dfrac{\text{energy}}{\text{charge}}$

ratio of two extensive properties is an intensive property.

SECTION–III : (Paragraph Type)

This section contains **2 paragraphs**. Based upon the first paragraph, **2 multiple choice questions** and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Questions 14 to 15

The concentration of potassium ions inside a biological cell is at least twenty times higher than the outside. The resulting potential difference across the cell is important in several processes such as transmission of nerve impulses and maintaining the ion balance. A simple model for such a concentration cell involving a metal M is :

$M(s)|M^+(aq\ ;\ 0.05\ \text{molar})\ \|\ M^+(aq\ ;\ 1\ \text{molar})|M(s)$

For the above electrolytic cell the magnitude of the cell potential $|E_{cell}| = 70$ mV.

14. For the above cell :-

(A) $E_{cell} < 0\ ;\ \Delta G > 0$ (B) $E_{cell} > 0\ ;\ \Delta G < 0$ (C) $E_{cell} < 0\ ;\ \Delta G^0 > 0$ (D) $E_{cell} > 0\ ;\ \Delta G^0 < 0$

Ans. (B)

Sol. The given cell is concentration cell.

$M(s) \longrightarrow M_L^+(aq) + e^-$ – at anode

$M_R^+(aq) + e^- \longrightarrow M(s)$ – at cathode

Net cell reaction : $M_R^+(aq) \longrightarrow M_L^+(aq)$

$$Q_{cell} = \frac{[M_L^+]}{[M_R^+]}$$

From Nernst equation : $E_{cell} = E_{cell} - \dfrac{0.59}{n}\log Q_{cell}$

$$= 0 - \frac{0.59}{1}\log\frac{[M_L^+]}{[M_R^+]} \qquad [\because E_{cell} = 0 \text{ for concentration cell.}]$$

$$= +0.059\ \log\frac{[M_R^+]}{[M_L^+]}$$

$$= 0.059\ \log\frac{1}{0.05}$$

so, E_{cell} = +ve and $\Delta G = -nFE_{cell}$ = –ve

15. If the 0.05 molar solution of M^+ is replaced by a 0.0025 molar M^+ solution, then the magnitude of the cell potential would be :-

(A) 35 mV (B) 70 mV (C) 140 mV (D) 700 mV

Ans. (C)

Sol. According to passage,

$0.059 \log 20 = 70$ mV

therefore, when 0.05 molar solution of M_L^+ is replaced by 0.0025 molar M_L^+ solution.

then,

$$E_{cell} = 0.059 \log \frac{[M_R^+]}{[M_L^+]}$$

$$= 0.059 \log \frac{1}{0.0025}$$

$$= 0.059 \log 400$$

$$= 0.059 \log 20^2$$

$$= 2 \times 0.059 \log 20$$

$$= 2 \times 70 = 140 \text{ mV}$$

Paragraph for Question 16 to 18

Copper is the most nobel of the first row transition metals and occurs in small deposits in several countries. Ores of copper include chalcanthite ($CuSO_4 . 5H_2O$), atacamite ($Cu_2Cl(OH)_3$), cuprite (Cu_2O), copper glance (Cu_2S) and malachite ($Cu_2(OH)_2CO_3$). However, 80% of the world copper production comes from the ore chalcopyrite ($CuFeS_2$). The extraction of copper from chalcopyrite involves partial roasting, removal of iron and self-reduction.

16. Partial roasting of chalcopyrite produces :-

(A) Cu_2S and FeO (B) Cu_2O and FeO (C) CuS and Fe_2O_3 (D) Cu_2O and Fe_2O_3

Ans. (A)

$$\underset{\text{Chalcopyrite}}{2CuFeS_2} + 4O_2 \xrightarrow{\text{Partial Roasting}} Cu_2S + 2FeO + 3SO_2$$

17. Iron is removed from chalcopyrite as :-

(A) FeO (B) FeS (C) Fe_2O_3 (D) $FeSiO_3$

Ans. (D)

FeO from by partial reduction is form slag with SiO_2 and removed from ore.

$FeO + SiO_2 \rightarrow FeSiO_3$

18. In self-reduction, the reducing species is :-

(A) S (B) O^{2-} (C) S^{2-} (D) SO_2

Ans. (C)

During self reduction of Cu_2S

$$Cu_2\overset{-2}{S} + 2Cu_2O \rightarrow 6Cu + \overset{+4}{S}O_2$$

In above process O.N. of S^{-2} is change to +4 (SO_2).

reducing species is = S^{-2}

SECTION–IV : (Integer Type)

This Section contains **TEN** questions. The answer to each question is a **single digit integer** ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

19. A student performs a titration with different burettes and finds titre values of 25.2 mL, 25.25 mL, and 25.0 mL. The number of significant figures in the average titre value is :-

Ans. 3

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Sol. Average $= \dfrac{25.2 + 25.25 + 25.0}{3} = 25.15$

The result cannot have more digits to the right of the decimal point than either of the original numbers.

$\therefore$ so result will be 25.2

$\therefore$ Number of significant figures in the result is 3.

20. The concentration of R in the reaction R $\rightarrow$ P was measured as a function of time and the following data is obtained:

[R] (molar)	1.0	0.76	0.40	0.10
t(min.)	0.0	0.05	0.12	0.18

The order of the reaction is.

Ans. 0

Sol. R $\longrightarrow$ P

Average rate in time interval (0 to 0.05 min.)

$$-\frac{\Delta[R]}{\Delta t} = -\frac{(0.75 - 1.0)}{0.05} = 5$$

Average rate in time interval (0 to 0.12 min.)

$$-\frac{\Delta[R]}{\Delta t} = -\frac{(0.4 - 1.0)}{0.12} = 5$$

Zero order reaction because rate is constant with time.

Alternative solution : By hit and trial method, assuming the reaction is of zero order, putting given data in integrated expression for zero order.

$[A_t] = [A_0] - kt$

At t = 0.05 min $\quad 0.75 = 1 - k \times 0.05 \Rightarrow k = 5$ (M/min)

At t = 0.12 min $\quad 0.4 = 1 - k \times 0.12 \Rightarrow k = 5$ (M/min)

the value of k is same from different data so reaction is zero order reaction.

21. The number of neutrons emitted when ${}^{235}_{92}U$ undergoes controlled nuclear fission to ${}^{142}_{54}Xe$ and ${}^{90}_{38}Sr$ is.

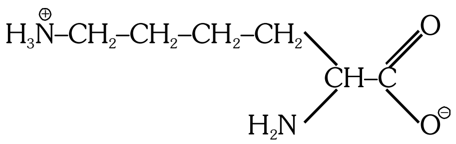
Ans. 4

Sol. Controlled nuclear fission of ${}^{235}_{92}U$ to ${}^{142}_{54}Xe$ and ${}^{90}_{38}Sr$ will take place as follows :

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{142}_{54}Xe + {}^{90}_{38}Sr + 4\,{}^{1}_{0}n$$

$\therefore$ number of emitted neutrons = 4

22. The total number of basic groups in the following form of lysine is :



Ans. 2

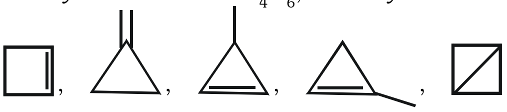
Sol. There are two basic groups in this form of lysine are

$-NH_2$ and $-\overset{O}{\overset{\|}{C}}-O^{\ominus}$

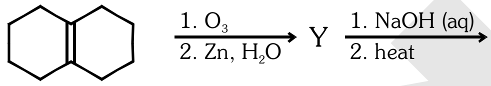
23. The total number of cyclic isomers possible for a hydrocarbon with the molecular formula C_4H_6 is.

Ans. 5

Sol. All cyclic isomers of C_4H_6, monocyclic and bicyclic are :

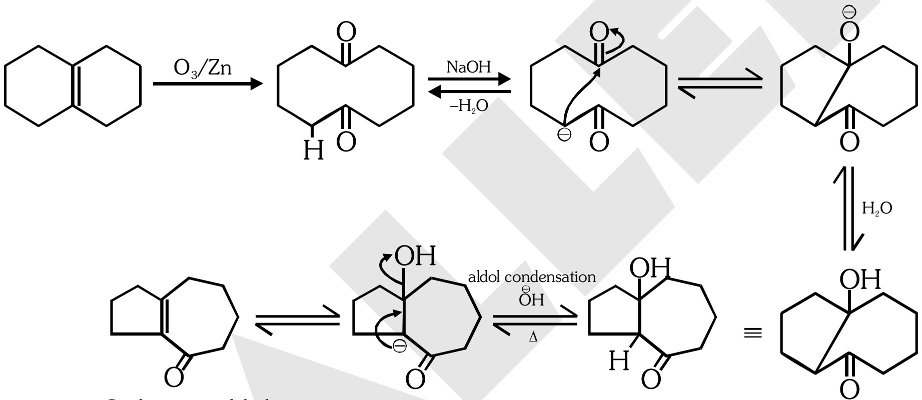


24. In the scheme given below, the total number of intramolecular aldol condensation products formed from 'Y' is :-



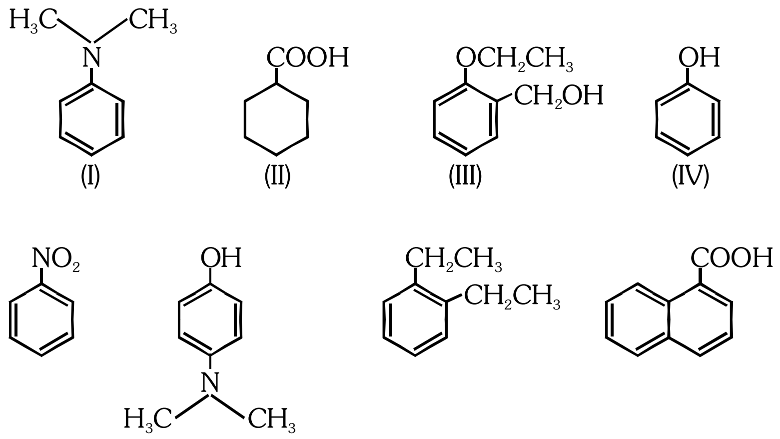
Ans. 1

Sol.



Only one aldol condensation product

25. Amongst the following, the total number of compounds soluble in aqueous NaOH is :



(I) (II) (III) (IV) (V) (VI) (VII) (VIII)

Ans. 5

II, III, IV, VI and VIII are soluble in aqueous NaOH.

26. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is :

KCN	K_2SO_4	$(NH_4)_2C_2O_4$	NaCl	$Zn(NO_3)_2$
$FeCl_3$	K_2CO_3	NH_4NO_3	LiCN	

Ans. 3

Basic solution turns red litmus paper blue.

Basic solution	Acidic solution	Neutral solution
1. KCN	1. $(NH_4)_2C_2O_4$	1. K_2SO_4
2. K_2CO_3	2. $Zn(NO_3)_2$	2. NaCl
3. LiCN	3. $FeCl_3$	
	4. NH_4NO_3	

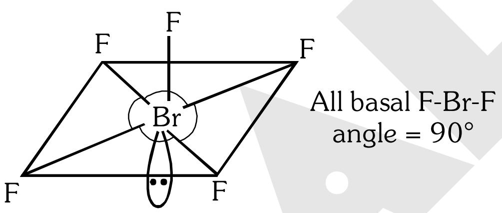
27. Based on VSEPR theory, the number of 90 degree F–Br–F angles in BrF_5 is.

Ans. 4

In all real AX_5E cases, the apical bond distance is less than the basal A–X bond distance. In addition among the AX_5E molecules, the apical - central - basal atom angle varies considerably. For example, it is 85 in BrF_5 and 92 in $XeOF_4$. Note that in BrF_5 the central atom is below the basal plane (presumably because of lone-pair repulsion of the basal bond electrons) even with this variation the basal - central - basal atom angle range only from 89.5 to 89.9 .

(Reference - Molecular origami : Precision scale models from paper. Robert M. Hanson)

However, such typical and complex through process do not seems to be examiners view point. Hence according to us, the bond angle should be taken as perfectly 90 by basic principles of VSEPR.



28. The value of n in the molecular formula $Be_nAl_2Si_6O_{18}$ is.

Ans. 3

$Be_nAl_2Si_6O_{18}$

O.N. of Be = +2

O.N. of Al = +3

O.N. of Si = +4

O.N. of O = –2

$n(+2) + 2(+3) + 6(+4) + 18(-2) = 0$

$2n + 6 + 24 - 36 = 0$

$2n - 6 = 0,\ 2n = 6,\ \ n = 3$

formula of silicate is $Be_3Al_2Si_6O_{18}$

PART - II (MATHEMATICS)

SECTION–I : (Single Correct Choice Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

29. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_2 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$ is

(A) $\frac{1}{18}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{36}$

Sol. Ans.(C)

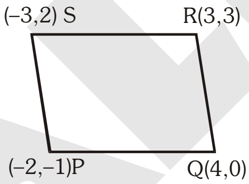
r_1, r_2, r_3 can be from the set (3, 6), (1, 4) or (2, 5) which can be done in $2 \times 2 \times 2 = 8$ ways and these can be arranged in 3! ways

$\therefore$ Probability $= \frac{3! \times 8}{216} = \frac{2}{9}$

30. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} - \hat{j}$, $4\hat{i}$, $3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a

(A) parallelogram, which is neither a rhombus nor a rectangle
(B) square
(C) rectangle, but not a square
(D) rhombus, but not a square

Sol. Ans. (A)



$\overrightarrow{PQ} = 6\hat{i} + \hat{j}$

$\overrightarrow{SR} = 6\hat{i} + \hat{j}$

$\therefore \quad \overrightarrow{PQ} = \overrightarrow{SR}$

$\overrightarrow{PS} = -\hat{i} + 3\hat{j}$

$\overrightarrow{QR} = -\hat{i} + 3\hat{j}$

$\therefore \quad \overrightarrow{PS} = \overrightarrow{QR}$

But $\overrightarrow{PQ}.\overrightarrow{PS} = -6 + 3 = -3 \neq 0$ & $|\overrightarrow{PQ}| \neq |\overrightarrow{PS}|$

$\Rightarrow$ PQRS is a parallelogram but neither a rhombus nor a rectangle.

31. The number of 3 × 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions, is

(A) 0 (B) $2^9 - 1$ (C) 168 (D) 2

Sol. Ans. (A)

The given matrix system is a linear system in x, y, z, hence it can have either a unique solution or no-solution or infinitely many solutions. It can never have exactly two distinct solutions.

32. The value of $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t \ell n(1+t)}{t^4 + 4} dt$ is

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{64}$

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Sol. **Ans. (B)**

Applying L-Hospital rule,

$$\lim_{x\to 0}\frac{\int_0^x \frac{t\ell n(1+t)}{t^4+4}dt}{x^3} = \lim_{x\to 0}\frac{\frac{x\ell n(1+x)}{x^4+4}}{3x^2}$$

$$= \lim_{x\to 0}\frac{\ell n(1+x)}{3x(x^4+4)} = \frac{1}{12}$$

33. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

Sol. **Ans. (B)**

$$\alpha^3+\beta^3 = q$$

$\Rightarrow$ $(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = q$

$\Rightarrow$ $\alpha\beta = \frac{p^3+q}{3p}$

sum of the roots = $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$$= \frac{p^2 - 2\left(\frac{p^3+q}{3p}\right)}{\frac{p^3+q}{3p}} = \frac{p^3-2q}{p^3+q}$$

Product of the roots = 1.

Required equation is

$(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

34. Let f, g and h be real-valued functions defined on the interval [0, 1] by $f(x) = e^{x^2}+e^{-x^2}$, $g(x) = xe^{x^2}+e^{-x^2}$ and $h(x) = x^2e^{x^2}+e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

(A) a = b and c ≠ b (B) a = c and a ≠ b (C) a ≠ b and c ≠ b (D) a = b = c

Sol. **Ans. (D)**

If $x \in [0, 1]$

then $x^2 \leq x \leq 1$

$$x^2e^{x^2} \leq xe^{x^2} \leq e^{x^2}$$

Add e^{-x^2} to all sides

$$x^2e^{x^2}+e^{-x^2} \leq xe^{x^2}+e^{-x^2} \leq e^{x^2}+e^{-x^2}$$

$\Rightarrow$ $h(x) \leq g(x) \leq f(x)$ .... (i)

where, $f(x) = e^{x^2}+e^{-x^2}$

$f'(x) = 2x(e^{x^2}-e^{-x^2}) > 0$

$\Rightarrow$ f(x) has a maxima at x = 1

$\Rightarrow$ $a = e + \frac{1}{e}$

$h(x) = x^2e^{x^2}+e^{-x^2}$

$$h'(x) = 2x^3e^{x^2}+2xe^{x^2}-2xe^{-x^2}$$

$= 2x^3e^{x^2} + 2x(e^{x^2} - e^{-x^2}) > 0$

$\Rightarrow$ h(x) has a maxima at x = 1

$\Rightarrow$ c = $e + \frac{1}{e}$

Now $\because$ $h(x) \le g(x) \le f(x)$

$\Rightarrow$ g(x) also has a maximum value at x = 1

$\Rightarrow$ a = b = c

35. If the angles A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$ is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$

Sol. Ans. (D)

A, B, C are in AP

$\Rightarrow$ 2B = A + C

$\Rightarrow$ A + B + C = π

$\Rightarrow$ 3B = π $\Rightarrow$ B = $\frac{\pi}{3}$

Now, $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$

$= \frac{a}{c}\, 2\sin C\cos C + \frac{c}{a}\, 2\sin A\cos A$

$= 2\left[\frac{a}{c}\frac{c}{2R}\cos C + \frac{c}{a}\frac{a}{2R}\cos A\right]$

$= \frac{1}{R}\left[a\cos C + c\cos A\right] = \frac{b}{R}$ (By projection formula) = 2 sinB

$= 2\sin\frac{\pi}{3} = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

36. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(A) x + 2y – 2z = 0 (B) 3x + 2y – 2z = 0 (C) x – 2y + z = 0 (D) 5x + 2y – 4z = 0

Sol. Ans. (C)

Normal vector to the plane containing the lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

$$\hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$$

Let direction ratios of required plane be a, b, c.

Now 8a – b – 10c = 0

and 2a + 3b + 4c = 0 ($\because$ plane contains the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$)

$\Rightarrow$ $\frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$

$\Rightarrow$ equation of plane is $x - 2y + z = d$

$\because$ plane contains the line, which passes through origin, hence origin lies on a plane.

$\Rightarrow$ equation of required plane is $x - 2y + z = 0$.

SECTION–II : (Multiple Correct Choice Type)

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE OR MORE** may be correct.

37. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t)z_1 + tz_2$ for some real number t with $0 < t < 1$. If Arg(w) denotes the principal argument of a nonzero complex number w, then

(A) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

(B) $\text{Arg}(z - z_1) = \text{Arg}(z - z_2)$

(C) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

(D) $\text{Arg}(z - z_1) = \text{Arg}(z_2 - z_1)$

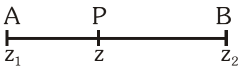
Sol. Ans.(A,C,D)

$z = z_1 + t(z_2 - z_1)$

$\dfrac{z - z_1}{z_2 - z_1} = t$, $t \in (0,1) \Rightarrow z = \dfrac{z_1(1-t) + tz_2}{(1-t) + t}$

point P(z) divides point $A(z_1)$ & $B(z_2)$ internally in ratio $(1 - t) : t$

Hence locus is a line segment such that P(z) lies between $A(z_1)$ & $B(z_2)$ as shown in figure.



Hence options A,C & D are correct.

38. The value(s) of $\displaystyle\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is (are)

(A) $\dfrac{22}{7} - \pi$ (B) $\dfrac{2}{105}$ (C) 0 (D) $\dfrac{71}{15} - \dfrac{3\pi}{2}$

Sol. Ans. (A)

$$I = \int_0^1 \frac{x^4\left(1-2x+x^2\right)^2}{1+x^2}dx$$

$$I = \int_0^1 \frac{x^4\left\{\left(1+x^2\right)^2 - 4x\left(1+x^2\right) + 4x^2\right\}}{1+x^2}dx$$

$$= \int_0^1 \left(1+x^2\right)x^4dx - \int_0^1 4x^5dx + 4\int_0^1 \frac{\left(x^6+1\right)-1}{1+x^2}dx$$

$$= \frac{1}{5} + \frac{1}{7} - 4.\frac{1}{6} + 4\int_0^1 \frac{\left(x^2+1\right)^3 - 3x^2\left(1+x^2\right)}{1+x^2}dx - 4\int_0^1 \frac{dx}{1+x^2}$$

$$= \frac{12}{35} - \frac{2}{3} + 4\int_0^1 \left(x^4 + 2x^2 + 1\right)dx - 12\int_0^1 x^2dx - \pi$$

$$= \frac{12}{35} - \frac{2}{3} + 4\left(\frac{1}{5} + \frac{2}{3} + 1\right) - 4 - \pi$$

$$= \frac{12}{35} - \frac{2}{3} + \frac{52}{15} - \pi$$

$$= \frac{22}{7} - \pi$$

39. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are)

(A) $-(2+\sqrt{3})$ (B) $1+\sqrt{3}$ (C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

Sol. Ans. (B)

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\frac{\sqrt{3}}{2} = \frac{(x^2+x+1)^2+(x^2-1)^2-(2x+1)^2}{2(x^2+x+1)(x^2-1)}$$

$$\Rightarrow \quad x^4(2-\sqrt{3})+x^3(2-\sqrt{3})-3x^2-x(2-\sqrt{3})+(\sqrt{3}+1)=0$$

$$\Rightarrow \quad (x^2-1)[(2-\sqrt{3})x^2+(2-\sqrt{3})x-(\sqrt{3}+1)]=0$$

Now $\begin{cases} x^2+x+1+x^2-1 > 2x+1 \\ x^2+x+1+2x+1 > x^2-1 \\ 2x+1+x^2-1 > x^2+x+1 \end{cases}$ ($\because$ sum of two sides is greater than third side)

$\Rightarrow \quad x > 1$

$\Rightarrow \quad x = 1+\sqrt{3}$

Alternate :

$$\tan\frac{\pi}{12} = \sqrt{\frac{(s-b)(s-a)}{s(s-c)}}$$

$$s = x^2+\frac{3x+1}{2},\quad s-b = \frac{3(x+1)}{2}, s-a = \frac{x-1}{2}, s-c = x^2-\frac{(x+1)}{2}$$

$$\tan\frac{\pi}{12} = \sqrt{\frac{\frac{3}{2}(x+1)\frac{(x-1)}{2}}{\left(x^2+\frac{3x+1}{2}\right)\left(x^2-\frac{x+1}{2}\right)}}$$

Simiplying

$$2-\sqrt{3} = \frac{\sqrt{3}}{2x+1}$$

$$x = \sqrt{3}+1$$

40. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be -

(A) $-\frac{1}{r}$ (B) $\frac{1}{r}$ (C) $\frac{2}{r}$ (D) $-\frac{2}{r}$

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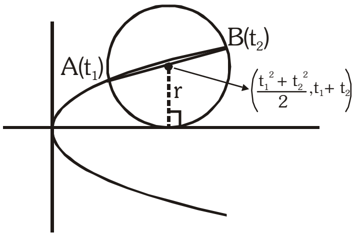
Sol. **Ans. (C,D)**



$t_1 + t_2 = r$

$\frac{2}{r} = \frac{2}{t_1+t_2}$

similarly $-\frac{2}{r}$ is also possible

41. Let f be a real-valued function defined on the interval $(0, \infty)$ by $f(x) = \ell nx + \int_0^x \sqrt{1+\sin t}\,dt$. Then which of the following statement(s) is (are) true?

(A) $f''(x)$ exists for all $x \in (0, \infty)$

(B) $f'(x)$ exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$

(C) there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (\alpha, \infty)$

(D) there exists $\beta > 0$ such that $|f(x)|+|f'(x)| \le \beta$ for all $x \in (0, \infty)$

Sol. **Ans. (B,C)**

$f(x) = \ell nx + \int_0^x \sqrt{1+\sin t}\,dt$

$f'(x) = \frac{1}{x} + \sqrt{1+\sin x}$

$f'(x) = \frac{1}{x} + \sqrt{2}\left|\cos\left(\frac{x}{2}-\frac{\pi}{4}\right)\right|$

$\because \left|\cos\left(\frac{x}{2}-\frac{\pi}{4}\right)\right|$ is non-derivable

$\therefore$ f '(x) is non-derivable but continuous.

hence option (A) is incorrect & option (B) is correct.

For option C

$f(x) = (\ell nx) + \int_0^x \left(\sqrt{1+\sin x}\right)dx$

since f(x) is positive increasing function for all x > 1

$\Rightarrow |f(x)| = f(x)$ & $|f'(x)| = f'(x)$

Let f(x) = y

$f'(x) - f(x) = \frac{1}{x} - \ell nx + \sqrt{1+\sin x} - \int_0^x \sqrt{1+\sin t}\,dt$

$f'(x) - f(x) = \frac{1}{x} - \ell nx + \sqrt{1+\sin x} - \sqrt{2}\int_0^x \left|\cos\left(\frac{t}{2}-\frac{\pi}{4}\right)\right|dt$

$\frac{1}{x} - \ell nx < 0$; when $\alpha > e$

$0 \le \sqrt{1+\sin x} \le \sqrt{2}$.

$\int_0^x \left|\cos\left(\frac{t}{2}-\frac{\pi}{4}\right)\right|dt > \sqrt{2}\ \forall \alpha > \frac{3\pi}{2}$

$\Rightarrow f'(x) - f(x) < 0\ \forall\ \alpha > \frac{3\pi}{2} > 1$

Hence option (C) is correct.
For option (D) $|f(x)| + |f'(x)| \to \infty$ when $x \to \infty$. Therefore option (D) is incorrect.

Alternate :

$$f(x) = \ell nx + \int_0^x \sqrt{1+\sin t}\,dt$$

$$f'(x) = \frac{1}{x} + \sqrt{1+\sin x} \qquad \text{........(i)}$$

for $x > 1$

$$\frac{1}{x} + \sqrt{1+\sin x} < 1 + \sqrt{2}$$

but $\ell nx + \int_0^x \sqrt{1+\sin t}\,dt$ will always be more than $1+\sqrt{2}$ for some $\alpha > 1$

$\because \int_0^x \sqrt{1+\sin t} > 0$ & ℓnx is increasing in $(1, \infty)$

$\Rightarrow$ $f(x) > f'(x)\ \forall\ \alpha > 1$

$\therefore$ (C) is correct

$$f''(x) = -\frac{1}{x^2} + \frac{\cos x}{2\sqrt{1+\sin x}}$$

$\Rightarrow$ f' is not derivable on $(0, \infty)$

at $\frac{3\pi}{2}, \frac{7\pi}{2}$

$\therefore$ (B) is also correct

f(x) is unbounded near x = 0 in (0, 1) hence $|f(x)|$ can never be made less than a finite number hence $|f(x)|+|f'(x)|$ can never be less than β.

SECTION–III : (Paragraph Type)

This section contains **2 paragraphs**. Based upon the first paragraph, **2 multiple choice questions** and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question 42 and 43

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

42. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is -

(A) $2x - \sqrt{5}y - 20 = 0$ (B) $2x - \sqrt{5}y + 4 = 0$ (C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

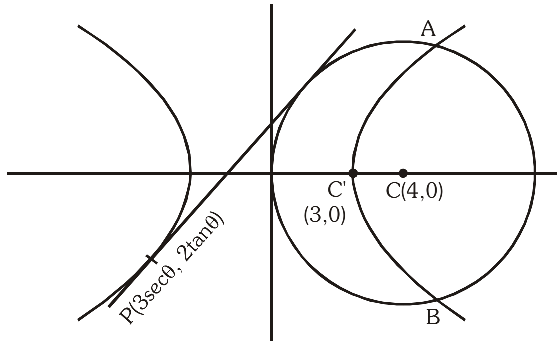
Sol. Ans. (B)

Let the point on the hyperbola $P(3\sec\theta, 2\tan\theta)$

Equation of tangent $\frac{x\sec\theta}{3} - \frac{y\tan\theta}{2} = 1$

$|p| = r$

$$\frac{\left|\frac{4}{3}\sec\theta - 1\right|}{\sqrt{\frac{\sec^2\theta}{9} + \frac{\tan^2\theta}{4}}} = 4$$



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$\Rightarrow \quad \frac{16}{9}\sec^2\theta + 1 - \frac{8}{3}\sec\theta = 16\left(\frac{4\sec^2\theta + 9\tan^2\theta}{4\times 9}\right)$

$16\sec^2\theta + 9 - 24\sec\theta = 52\sec^2\theta - 36$

$\Rightarrow \quad 36\sec^2\theta + 24\sec\theta - 45 = 0$

$\Rightarrow \quad 12\sec^2\theta + 8\sec\theta - 15 = 0$

$\Rightarrow \quad 12\sec^2\theta + 18\sec\theta - 10\sec\theta - 15 = 0$

$\Rightarrow \quad (6\sec\theta - 5)(2\sec\theta + 3) = 0$

$\sec\theta = \frac{5}{6}$ (not possible), $\sec\theta = -\frac{3}{2}$

$\tan\theta = \pm\sqrt{\frac{9}{4} - 1} = \pm\frac{\sqrt{5}}{2}$ ($\because$ slope is positive $\Rightarrow \tan\theta = -\frac{\sqrt{5}}{2}$)

Hence the required equation be $-\frac{3x}{2\times 3} + \frac{y\sqrt{5}}{2\times 2} = 1 \quad \Rightarrow \quad 2x - \sqrt{5}y + 4 = 0$

43. Equation of the circle with AB as its diameter is -

(A) $x^2 + y^2 - 12x + 24 = 0$
(B) $x^2 + y^2 + 12x + 24 = 0$
(C) $x^2 + y^2 + 24x - 12 = 0$
(D) $x^2 + y^2 - 24x - 12 = 0$

Sol. Ans. (A)

Solving (a) & (b) for x, we get

$x = 6$

$y = \pm 2\sqrt{3}$

$(x - 6)^2 + y^2 - 12 = 0$

$x^2 + y^2 - 12x + 24 = 0$

Option (A) is correct

Paragraph for Question 44 to 46

Let p be an odd prime number and T_p be the following set of 2 × 2 matrices :

$$T_p = \left\{A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots, p-1\}\right\}$$

44. The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det (A) divisible by p is -

(A) $(p - 1)^2$
(B) $2(p - 1)$
(C) $(p - 1)^2 + 1$
(D) $2p - 1$

Sol. Ans. (D)

If A is symmetric, $A^T = A$

$\Rightarrow \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} a & c \\ b & a \end{bmatrix}$

$\Rightarrow \quad b = c$

If A is skew symmetric, $A^T = -A$

$\Rightarrow \begin{bmatrix} a & b \\ c & a \end{bmatrix} = \begin{bmatrix} -a & -c \\ -b & -a \end{bmatrix}$

$\Rightarrow \quad a = 0, \; b + c = 0$

$\because \quad b, c \geq 0 \Rightarrow \quad a = 0, b = 0, c = 0$

Now, $\det(A) = a^2 - bc$

$= a^2 - b^2$ ($\because$ b = c for A being symmetric or skew symmetric or both)

$= (a - b)(a + b)$ is divisible by p.

Let $(a - b)(a + b) = \lambda p, \; \lambda \in I$

Range of (a + b) is 0 to 2p − 2 which includes only one multiple of p i.e. p

$\therefore$ a + b = p & a − b $\in$ I

$\Rightarrow$ possible number of pairs of a & b will be p − 1.

Also, range of (a − b) is 1 − p to p − 1 which includes only one multiple of p i.e. 0

$\therefore$ a − b = 0 & a + b $\in$ I

$\Rightarrow$ Possible number of pairs of a & b will be p.

Hence total number of A in T_p will be p + p − 1 = 2p − 1

45. The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is -

[**Note** : The trace of a matrix is the sum of its diagonal entries.]

(A) $(p-1)(p^2-p+1)$ (B) $p^3-(p-1)^2$ (C) $(p-1)^2$ (D) $(p-1)(p^2-2)$

Sol. Ans. (C)

46. The number of A in T_p such that det (A) is not divisible by p is -

(A) $2p^2$ (B) p^3-5p (C) p^3-3p (D) p^3-p^2

Sol. Ans. (D)

Total number of A in $T_P = p^3$

From Q.45 when a $\neq$ 0 & det(A) is divisible by p, then number of A will be $(p-1)^2$

When a = 0 & det(A) is divisible by p, then number of A will be 2p−1.

So, total number of A for which det(A) is divisible by p

$= (p-1)^2 + 2p - 1$

$= p^2$

So number of A for which det(A) is not divisible by p

$= p^3 - p^2$

SECTION–IV : (Integer Type)

This Section contains **TEN** questions. The answer to each question is a **single digit integer** ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

---

47. Let S_k, k = 1,2,.......,100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right|$ is

Sol. Ans. 3

$$S_k = \frac{\frac{k-1}{k!}}{1-\frac{1}{k}} = \frac{1}{(k-1)!} \qquad \text{for } k \geq 2,\ S_1 = 0$$

Now $\frac{100^2}{100!}+\sum_{k=2}^{100}\left|\left(k^2-3k+1\right)\cdot\frac{1}{(k-1)!}\right| + S_1$

$$\frac{100^2}{100!}+\frac{1}{1!}+\sum_{k=3}^{100}\left|\frac{1}{(k-3)!}-\frac{1}{(k-1)!}\right| + 0 \qquad \left(\because S_2=\frac{1}{1!}\right)$$

$$= \frac{100^2}{100!}+1+\left|\frac{1}{0!}-\frac{1}{2!}\right|+\left|\frac{1}{1!}-\frac{1}{3!}\right|+\left|\frac{1}{2!}-\frac{1}{4!}\right|+\ldots+\left|\frac{1}{97!}-\frac{1}{99!}\right|$$

$$= \frac{100^2}{100!}+3-\frac{1}{98!}-\frac{1}{99!}$$

$$= \frac{100^2}{100!}+3-\frac{100}{99!}=3$$

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48. The number of all possible values of θ, where $0 < \theta < \pi$, for which the system of equations

$$(y + z)\cos 3\theta = (xyz)\sin 3\theta$$

$$x\sin 3\theta = \frac{2\cos 3\theta}{y} + \frac{2\sin 3\theta}{z}$$

$$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$$

have a solution (x_0, y_0, z_0) with $y_0 z_0 \neq 0$, is

Sol. **Ans. 3**

$(y + z)\cos 3\theta = (xyz)\sin 3\theta$ ...(i)

$x\sin 3\theta = \dfrac{2\cos 3\theta}{y} + \dfrac{2\sin 3\theta}{z}$ ...(ii)

$(xyz)\sin 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$ ...(iii)

where $yz \neq 0$ and $0 < \theta < \pi$

from (i) & (iii)

$(y + z)\cos 3\theta = (y + 2z)\cos 3\theta + y\sin 3\theta$

$\Rightarrow \quad z\cos 3\theta + y\sin 3\theta = 0$ ...(iv)

from eq^n (ii)

$2z\cos 3\theta + 2y\sin 3\theta = xyz\sin 3\theta$ ...(v)

from equation (iv) & (v)

$\Rightarrow \quad xyz\sin 3\theta = 0$

$\Rightarrow \quad x\sin 3\theta = 0 \quad$ as $yz \neq 0$

Possible cases are either $x = 0$ or $\sin 3\theta = 0$

Case (1) : if $x = 0$

$\Rightarrow \quad y + z = 0 \Rightarrow y = -z$

from eq^n (iv) $\cos 3\theta = \sin 3\theta$

$\Rightarrow \quad 3\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}$

$\Rightarrow \quad \theta = \dfrac{\pi}{12}, \dfrac{5\pi}{12}, \dfrac{9\pi}{12}$

Case (2) : if $\sin 3\theta = 0$

$\Rightarrow \quad \theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

But these values does not satisfy given equations.

Hence, total number of possible values of θ are 3.

49. Let f be a real-valued differentiable function on R (the set of all real numbers) such that f(1) = 1. If the y-intercept of the tangent at any point P(x,y) on the curve y = f(x) is equal to the cube of the abscissa of P, then the value of f(–3) is equal to

Sol. **Ans. 9**

Given $y = f(x)$

Tangent at point P(x, y)

$Y - y = \left(\dfrac{dy}{dx}\right)_{(x,y)}(X - x)$

Now y–intercept $\Rightarrow Y = y - x\dfrac{dy}{dx}$

Given that, $y - x\dfrac{dy}{dx} = x^3$

$\Rightarrow \quad \dfrac{dy}{dx} - \dfrac{y}{x} = -x^2$ is a linear differential equation with I.F. = $e^{\int -\frac{1}{x}dx} = e^{-\ell nx} = e^{\ell n\left(\frac{1}{x}\right)} = \frac{1}{x}$

Hence, solution is $\frac{y}{x} = \int -x^2.\frac{1}{x}dx + C$

or $\frac{y}{x} = -\frac{x^2}{2} + C$

Given $f(1) = 1$

Substituing we get, $C = \frac{3}{2}$

so $y = -\frac{x^3}{2} + \frac{3}{2}x$

Now $f(-3) = \frac{27}{2} - \frac{9}{2} = 9$

50. The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ such that $\theta \neq \frac{n\pi}{5}$ for n = 0, ±1, ±2 and tanθ = cot 5θ as well as sin 2θ = cos4θ is

Sol. Ans. 3

$\sin 2\theta = \cos 4\theta$

$\sin 2\theta = 1 - 2\sin^2 2\theta$

$2\sin^2 2\theta + \sin 2\theta - 1 = 0$

$\Rightarrow \sin 2\theta = -1, \frac{1}{2}$

$\therefore \theta = -\frac{\pi}{4}, \frac{\pi}{12}, \frac{5\pi}{12}$

Now $\tan\theta = \cot 5\theta$ (Given)

All three obtained values of θ satisy the given equation.

Hence, number of values of θ are 3.

51. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

Sol. Ans. 2

$$y = \frac{1}{\frac{(1-\cos 2\theta)}{2} + \frac{3\sin 2\theta}{2} + 5\frac{(1+\cos 2\theta)}{2}} = \frac{1}{3 + \left(2\cos 2\theta + \frac{3\sin 2\theta}{2}\right)}$$

$$y = \frac{2}{6 + (4\cos 2\theta + 3\sin 2\theta)}$$

$$y_{max} = \frac{2}{6-5} = 2$$ (Since $(-5 \le 4\cos 2\theta + 3\sin 2\theta \le 5)$)

52. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $\left(2\vec{a} + \vec{b}\right).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} - 2\vec{b}\right)\right]$ is

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Sol. Ans. 5

$(2\vec{a}+\vec{b}).[(\vec{a}\times\vec{b})\times\vec{a}-2(\vec{a}\times\vec{b})\times\vec{b}]$

$= (2\vec{a}+\vec{b}).[a^2\vec{b}-(\vec{a}.\vec{b})\vec{a}-2\{(\vec{a}.\vec{b})\vec{b}-b^2\vec{a}\}]$

$= (2\vec{a}+\vec{b}).[a^2\vec{b}+2b^2\vec{a}]$; as $\vec{a}.\vec{b}=0$

$= (2\vec{a}+\vec{b}).[2\vec{a}+\vec{b}]$ as $[a^2 = b^2 = 1]$

$\Rightarrow 4a^2 + b^2 = 5$

53. The line $2x + y = 1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

Sol. Ans. 2

As directrix cut the x–axis at $(\pm a/e, 0)$

Hence, $\frac{2a}{e} + 0 = 1$ (for nearer directrix)

$\Rightarrow \quad 2a = e \qquad$ ...(i)

Now, $b^2 = a^2(e^2 - 1) = a^2(4a^2 - 1)$

$\Rightarrow \quad \frac{b^2}{a^2} = 4a^2 - 1 \qquad$ ...(ii)

Given line $y = -2x + 1$ is a tangent to the hyperbola

condition of tangency is $c^2 = a^2m^2 - b^2$

$\Rightarrow \quad 1 = 4a^2 - b^2$

$\Rightarrow \quad 4a^2 - 1 = b^2 \qquad$ ...(iii)

from (ii) & (iii), $a^2 = 1$

$\Rightarrow$ from (ii), $b^2 = 3$

$\Rightarrow \quad e = \sqrt{\frac{1+3}{1}} = 2$

54. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is

Sol. Ans. 6

Plane containing the line

Direction ratio's of normal to the plane : $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 3 & 4 & 5\end{vmatrix} = -\hat{i}+2\hat{j}-\hat{k}$

Hence equation of plane $1(x - 1) - 2(y - 2) + 1(z - 3) = 0$

i.e. $x - 2y + z = 0$

As given plane must be parallel $\Rightarrow A = 1$

& distance between the planes $\left|\frac{d-0}{\sqrt{1^2+2^2+1^2}}\right| = \sqrt{6}$

$|d| = 6$

55. For any real number x, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by

$$f(x) = \begin{cases} x-[x] & \text{if } [x] \text{ is odd,} \\ 1+[x]-x & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\, dx$ is

Sol. Ans. 4

$$f(x) = \begin{cases} \{x\} & \text{when } -9 \le x < -8; -7 \le x < -6, \ldots\ldots.. \\ 1-\{x\} & \text{when } -10 \le x \le -9; -8 \le x < -7, \ldots\ldots \end{cases}$$

Since f(x) & cosπx both are periodic functions having period 2.

$$I = \frac{10\times\pi^2}{10}\left(\int_0^1 (1-\{x\})\cos \pi x dx + \int_1^2 \{x\}\cos \pi x dx\right)$$

$$= \pi^2\left(\int_0^1 (1-x)\cos \pi x dx + \int_1^2 (x-1)\cos \pi x dx\right)$$

$$= \pi^2\left(\int_0^1 \cos \pi x dx - \int_1^2 \cos \pi x dx + \int_1^2 x\cos \pi x dx - \int_0^1 x\cos \pi x dx\right)$$

$\Rightarrow \quad I = 4$

56. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of distinct complex numbers z satisfying

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$ is equal to

Sol. Ans. 1

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

$$\Rightarrow \quad z = 0 \quad \text{or} \quad \begin{vmatrix} 1 & 1 & 1 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \quad \Rightarrow \quad C_2 \to C_2 - C_1 \ \& \ C_3 \to C_3 - C_1$$

$$\begin{vmatrix} 1 & 0 & 0 \\ \omega & z+(\omega^2-\omega) & 1-\omega \\ \omega^2 & (1-\omega^2) & z-(\omega^2-\omega) \end{vmatrix} = 0$$

$z^2 - (\omega^2 - \omega)^2 - (1 - \omega)(1 - \omega^2) = 0$

$z^2 - \omega - \omega^2 + 2 - 1 + \omega + \omega^2 - 1 = 0$

$z^2 = 0 \Rightarrow z = 0$

Number of values is 1

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PART-III (PHYSICS)

SECTION–I : (Single Correct Choice Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct.**

57. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistance R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is

(A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ (B) $R_{100} = R_{40} + R_{60}$ (C) $R_{100} > R_{60} > R_{40}$ (D) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

Ans. (D)

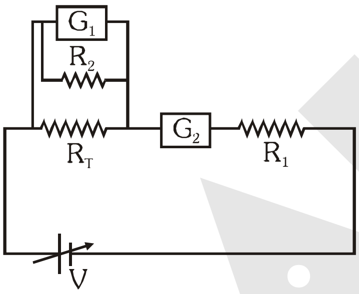
Sol. $P = \frac{V^2}{R}$ and $100W > 60W > 40W \Rightarrow \frac{V^2}{R_{100}} > \frac{V^2}{R_{60}} > \frac{V^2}{R_{40}} \Rightarrow \frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

[**Note** : Although 100 = 60 +40 so at room tempeature

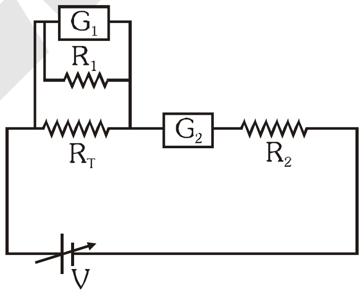
$\frac{V^2}{R_{100}} = \frac{V^2}{R_{60}} + \frac{V^2}{R_{40}} \Rightarrow \frac{1}{R_{100}} = \frac{1}{R_{60}} + \frac{1}{R_{40}}$ (Applicable only at room temperature)]

58. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is :–

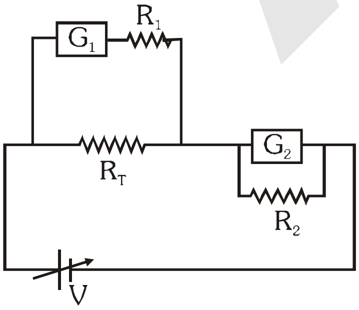
(A)



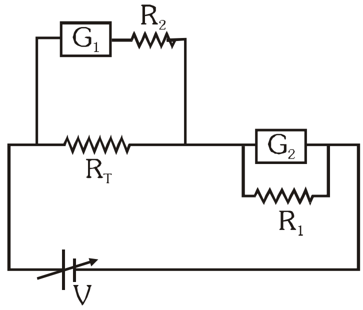
(B)



(C)

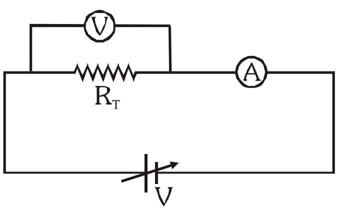


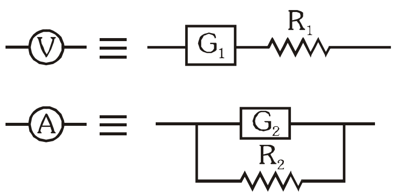
(D)



Ans. (C)

Sol.



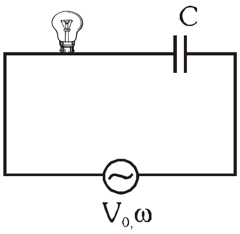


59. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

(A) the bulb glows dimmer
(B) the bulb glows brighter
(C) total impedance of the circuit is unchanged
(D) total impedance of the circuit increases

Ans. (B)

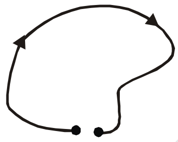
Sol.

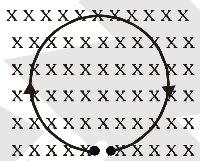


$$I = \frac{V_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

If $\omega\uparrow$ then $I\uparrow$ and $P = I^2R$ so the bulb glows brighter.

60. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is

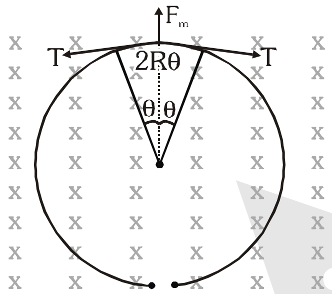




(A) IBL
(B) $\frac{IBL}{\pi}$
(C) $\frac{IBL}{2\pi}$
(D) $\frac{IBL}{4\pi}$

Ans. (C)

Sol.

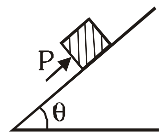


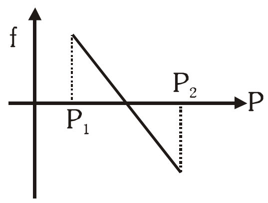
$2T\sin\theta = IB(2R\theta)$.

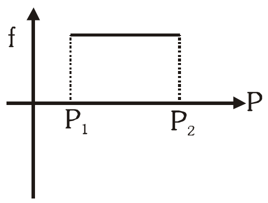
As θ is small so $\sin\theta \approx \theta$

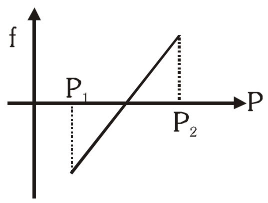
$\Rightarrow 2T(\theta) = 2IBR\theta \Rightarrow T = BIR = \frac{IBL}{2\pi}$ $[\because 2\pi R = L]$

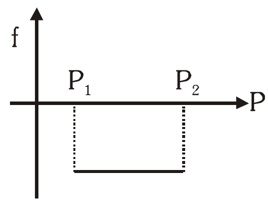
61. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like



(A)


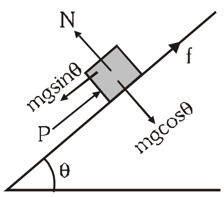
(B)


(C)


(D)


Ans. (A)

Sol.



f varies from $\mu mg\cos\theta$ to $-\mu mg\cos\theta$.

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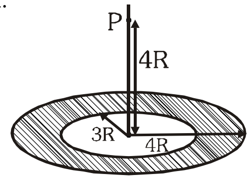
62. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is

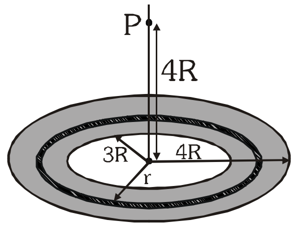


(A) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (B) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}(\sqrt{2}-1)$

Ans. **(A)**

Sol. Mass / Area of disk (σ) $=\frac{M}{\pi(4R)^2-(3R)^2}=\frac{M}{7\pi R^2}$



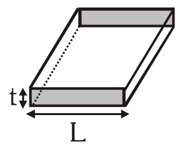
Mass of ring dm = (σ) (2πrdr)

$$U=\int dU=-\int\frac{G(dm)(1)}{\sqrt{16R^2+r^2}}=-\int_{3R}^{4R}\frac{2\pi G\sigma rdr}{\sqrt{16R^2+r^2}}$$

Put r =4R tanθ and solve, we get $\Delta U=-\frac{2GM(4\sqrt{2}-5)}{7R}$

Work done by the external agent = $U_\infty - U = \frac{2GM(4\sqrt{2}-5)}{7R}$

63. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is



(A) directly proportional to L (B) directly proportional to t

(C) independent of L (D) independent of t

Ans. **(C)**

Sol. $R=\frac{\rho L}{A}=\frac{\rho L}{Lt}=\frac{\rho}{t}\Rightarrow$ independent of L

64. A real gas behaves like an ideal gas if its

(A) pressure and temperature are both high (B) pressure and temperature are both low

(C) pressure is high and temperature is low (D) pressure is low and temperature is high

Ans. **(D)**

SECTION–II : (Multiple Correct Choice Type)

This section contains **5 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** may be correct.

65. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses?

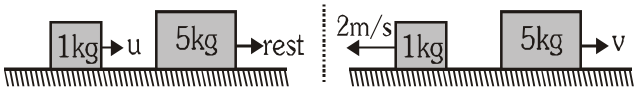
(A) Total momentum of the system is 3 kg ms^{-1}

(B) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

(C) Kinetic energy of the centre of mass is 0.75 J

(D) Total kinetic energy of the system is 4J

Ans. (AC)

Sol. 

Conservation of linear momentum $(1)\ u = -(1)\ 2 + (5)\ v \Rightarrow 5v - 2 = u$ ... (i)

By definition of 'e' $1 = \frac{v+2}{u} \Rightarrow v + 2 = u$ ... (ii)

By solving above equations v=1 ms^{-1} and u = 3 ms^{-1}

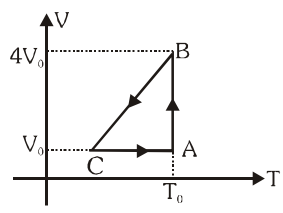
For (A) : Total momentum of system = $1 \times u = 3$ kg ms^{-1}

For (B) : Momentum of 5 kg after collision = 5 (1) = 5 kg ms^{-1}

For (C) : $K_{cm} = \frac{1}{2}(1+5)\left(\frac{1\times 3+0}{1+5}\right) = 0.75J$

For (D) : Total kinetic energy of the system = $\frac{1}{2}(1)(3)^2 = 4.5J$

66. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following



(A) Internal energies at A and B are the same

(B) Work done by the gas in process AB is $P_0V_0\ \ell n\ 4$

(C) Pressure at C is $\frac{P_0}{4}$

(D) Temperature at C is $\frac{T_0}{4}$

Ans. (AB)

Sol. $T_A = T_B \Rightarrow$ Internal energies are equal and work done in process AB = $P_0V_0\ell n\left(\frac{4V_0}{V_0}\right) = P_0V_0\ell n(4)$

Note : Had the process BC been isobaric then answer would have been **(ABCD).** But the question does not clearly indicate whether BC passes through **ORIGIN or NOT.**

67. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 sec for this and records 40 seconds for 20 oscillations. For this observation, which of the following statement(s) is (are) true?

(A) Error ΔT in measuring T, the time period is 0.05 seconds

(B) Error ΔT in measuring T, the time period is 1 second

(C) Percentage error in the determination of g is 5%

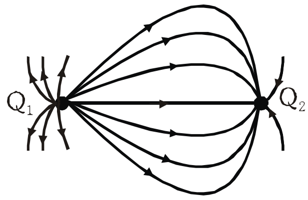
(D) Percentage error in the determination of g is 2.5%

Ans. (AC)

Sol. Error in $\Delta T = \frac{1\,\text{sec}}{20} = 0.05\,\text{sec}$

$$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g \propto \frac{\ell}{T^2} \Rightarrow \frac{\Delta g}{g} = \pm\left(\frac{2\Delta T}{T} + \frac{\Delta \ell}{\ell}\right) \Rightarrow 100 \times \frac{\Delta g}{g} = \pm\left(\frac{2\times 0.05}{2} + \frac{0}{1}\right)\times 100 = \pm 5\%$$

68. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x–axis are shown in the figure. These lines suggest that



(A) $|Q_1| > |Q_2|$

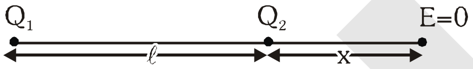
(B) $|Q_1| < |Q_2|$

(C) at a finite distance to the left of Q_1 the electric field is zero

(D) at a finite distance to the right of Q_2 the electric field is zero

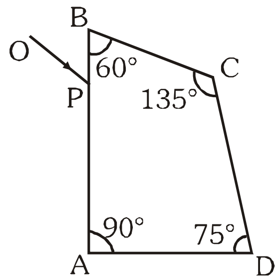
Ans. (AD)

Sol. Number of lines of force exiting from Q_1 is greater than that of enetering in $Q_2 \Rightarrow |Q_1| > |Q_2|$



$\frac{KQ_1}{(\ell + x)^2} = \frac{KQ_2}{(x^2)} \Rightarrow \frac{x}{(\ell + x)} = \sqrt{\frac{Q_1}{Q_2}}$ because $x > 0$ so there will be point in right of Q_2.

69. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60 (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?



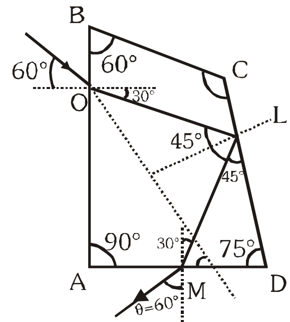
(A) The ray gets totally internally reflected at face CD

(B) The ray comes out through face AD

(C) The angle between the incident ray and the emergent ray is 90

(D) The angle between the incident ray and the emergent ray is 120

Ans. (ABC)



Sol. At point O (1) sin60 = $\sqrt{3}\sin r \Rightarrow$ r=30

At point L : TIR occurs : because $\sqrt{3}$ sin 45 > 1

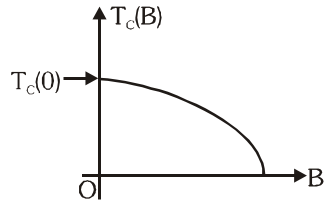
At point M : $\sqrt{3}$ sin (30) = $(1)\sin\theta \Rightarrow \theta = 60°$

SECTION–III : (Paragraph Type)

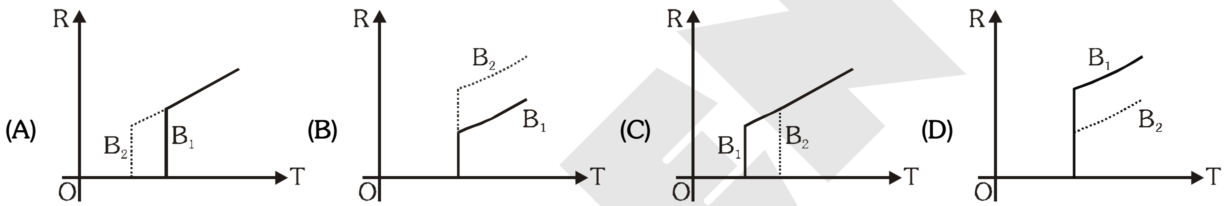
This section contains **2 paragraphs**. Based upon the first paragraph **2 multiple choice questions** and based upon the second paragraph **3 multiple choice questions** have to be answered. Eachof these questions has four choice (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

Paragraph for Questions 70 to 71

Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of T_C (B) on B is shown in the figure.



70. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1(sold line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?



Ans.(A)

Sol. Since T_C decreases as B increases therefore, if $B_2 > B_1$, then $T_{C2} < T_{C1}$

71. A superconductor has $T_C(0)$ = 100 K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when

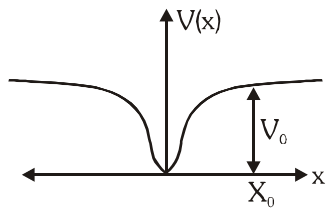
(A) B=5 Tesla, T_C (B) = 80 K
(B) B=5 Tesla, 75 K < T_C(B) < 100 K
(C) B= 10 Tesla, 75 K < T_C(B) < 100K
(D) B = 10 Tesla, T_C(B) = 70 K

Ans. (B)

Sol. Since T_C decreases as B increases therefore, if $B_2 > B_1$, then $T_{C2} < T_{C1}$

Paragraph for questions 72 to 74

When a particle of mass m moves on the x–axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x=0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x–axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for |x| near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure)



72. If the total energy of the particle is E, it will perform periodic motion only if

(A) E < 0
(B) E > 0
(C) V_0 > E > 0
(D) E > V_0

Ans. (C)

Sol. To perform peridodic motion, total energy must be less than, V_0

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73. For periodic motion of small amplitude A, the time period T of this particle is proportional to

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

Ans. (B)

Sol. Use dimensional analysis.

74. The acceleration of this particle for $|x| > X_0$ is

(A) proportional to V_0 (B) proportional to $\frac{V_0}{mX_0}$ (C) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (D) Zero

Ans. (D)

Sol. if $|x| > x_0$; $\frac{\partial V}{\partial x} = 0 \Rightarrow F = 0 \Rightarrow a = 0$

SECTION–IV : (Integer Type)

This section contains **TEN questions**. The answer to each question is a **single–digit integer**, ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

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75. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the Earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be

Ans. (3)

Sol. $v_e = \sqrt{2gR} \therefore v_e \propto \frac{g}{\sqrt{\rho}} \because \rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{g}{\frac{4\pi GR}{3}} \Rightarrow v_P = 3km/hr$

76. A piece of ice (heat capacity = 2100 J kg^{-1} C^{-1} and latent heat = 3.36×10^5 J kg^{-1}) of mass m grams is at –5 C at atmospheric pressure. It is given 420 J of heat so that the ice starts melting. Finally when the ice–water mixture is in equilibrium, it is found that 1 gm of ice has melted. Assuming there is no other heat exchange in the process, the value of m is

Ans. (8)

Sol. By principle of calorimetry $m \times 2100 \times 5 + 10^{-3} \times 3.36 \times 10^5 = 420 \therefore m = 8$ gm

77. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

Ans. (7)

Apparent frequency of sound reflected from car $f = \left(\frac{v + v_0}{v - v_0}\right)f_0 \approx \left(1 + \frac{2v_0}{v}\right)f_0$

$\because \frac{\Delta f}{f_0} \times 100 = 1.2 \therefore \frac{2v_0}{v} = \frac{1.2}{100} \Rightarrow v_0 = 2ms^{-1} = 2 \times \frac{18}{5}km/h \approx 7km/h$

78. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is

Ans. (6)

Sol. $m = \frac{f}{f + u} \Rightarrow m_{25} = \frac{20}{20 - 25} = -4$ and $m_{50} = \frac{20}{20 - 50} = -\frac{2}{3} \Rightarrow \frac{m_{25}}{m_{50}} = \frac{4 \times 3}{2} = 6$

79. An α–particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is

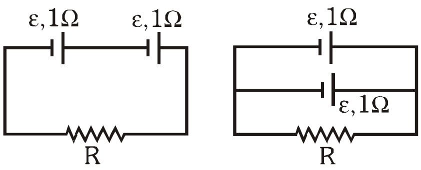
Ans. (3)

Sol. $\frac{h}{\lambda} = p = \sqrt{2mqV} \Rightarrow \frac{\lambda_P}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_P q_P}} = \sqrt{4\times 2} = \sqrt{8} \approx 3$

80. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$ then the value of R in Ω is

Ans. (4)

Sol.



$J_1 = \left(\frac{2\varepsilon}{R+2}\right)^2 R$ and $J_2 = \left(\frac{\varepsilon}{R+1/2}\right)^2 R$ as $\frac{J_1}{J_2} = 2.25$ so $\frac{4\varepsilon^2}{(R+2)} = 2.25\frac{4\varepsilon^2}{(1+2R)^2} \Rightarrow R = 4\Omega$

81. Two spherical bodies A (radius 6 cm) and B (radius 18 cm) are at temperatures T_1 and T_2, respectively. The maximum intensity in the emission spectrum of A is at 500 nm and in that of B is at 1500 nm. Considering then to be black bodies, what will be the ratio of the rate of total energy radiated by A to that of B?

Ans. (9)

Sol. $E = \sigma AT^4$ and $\lambda_m T = b$ so $E \propto \frac{R^2}{\lambda_m^4}$

82. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is

Ans. (5)

Sol. $A = \sqrt{A_1^2 + A_2^2}\left(\because\ \Delta\phi = \frac{\pi}{2}\right) \Rightarrow A = \sqrt{4^2+3^2} = 5$

83. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1m and its cross–sectional area is $4.9 \times 10^{-7}\ m^2$. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s^{-1}. If the Young's modulus of the material of the wire is $n \times 10^9\ Nm^{-2}$, the value of n is

Ans. (4)

Sol. $\frac{\text{stress}}{\text{strain}} = Y \Rightarrow F = \left(\frac{AY}{\ell}\right)x \Rightarrow \omega = \sqrt{\frac{AY}{m\ell}} \Rightarrow n = 4$

84. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is

Ans. (6)

Sol. $\frac{L}{L_B} = \frac{m_A v_A r_A + m_B v_B r_B}{m_B v_B r_B} = \frac{m_A v_A r_A}{m_B v_B r_B} + 1 = \frac{m_B}{m_A} + 1 = 6$

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